

Launch Vehicle Trajectory Simulation: Multi-Stage Burn Modeling and Orbital Insertion

Research Report — launch-veh-sim v1.0

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ABSTRACT

We present a numerical trajectory simulation framework for eleven launch vehicles, implemented in Go, that models multi-stage ascent through an exponential atmosphere with spherical gravity and J2 perturbation. The simulator introduces a multi-burn second-stage architecture with coast phases and apogee-restart circularization, addressing the key challenge of low-thrust-to-weight-ratio (TWR) upper stages that cannot follow simplistic pitch-table profiles. Validation against known orbital parameters shows correct LEO insertion for 6 of 11 vehicles (Falcon 9, Falcon 9 Expendable, Falcon Heavy, Starship, New Glenn, Long March 5), while identifying a systematic guidance deficit for vehicles with Centaur-class upper stages (Vulcan Centaur, Delta IV, Atlas V 551) and hydrogen-fueled second stages (Ariane 6). We demonstrate that pitch-table guidance fails for $TWR < 0.35$ stages and propose closed-loop guidance as the necessary evolution.

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1. Introduction

Orbital launch vehicle trajectory simulation is fundamental to mission design, range safety analysis, and vehicle performance verification. While high-fidelity tools like POST II and GMAT exist for mission-level design, there remains a need for rapid, transparent trajectory models that can be validated, inspected, and iterated quickly.

We present `launch-veh-sim`, a Go-based trajectory simulator that models the complete ascent-to-orbit sequence for eleven launch vehicles spanning American, European, and Chinese space programs. The simulator implements a multi-burn second-stage architecture with coast phases and apogee-restart circularization, a capability essential for realistic modeling of modern upper stages.

The key contribution of this work is twofold: first, a validated physics model for multi-stage ascent with aerodynamic drag, altitude-dependent thrust/ISP, and spherical gravity; and second, a systematic identification of the pitch-table guidance regime boundary,

demonstrating that stages with $TWR < 0.35$ require closed-loop guidance to achieve stable orbit insertion.

2. Vehicle Catalog

2.1 Overview

The simulator models eleven launch vehicle configurations, ranging from medium-lift vehicles like Atlas V 551 to super-heavy lifters like Starship. The following table summarizes the fleet:

VEHICLE	ORIGIN	STAGES	BOOSTERS	PAYLOAD (KG)	INCL. (°)	MULTI-BURN
Falcon 9 Block 5	USA	2	—	15,000	28.5	Yes
Falcon 9 (Expendable)	USA	2	—	18,000	28.5	Yes
Falcon Heavy	USA	2	2× F9 Booster	26,000	28.5	Yes
Starship	USA	2	—	150,000	28.5	Yes
New Glenn	USA	2	—	6,100	49.4	Yes
Vulcan Centaur (VC4)	USA	2	4× GEM-63XL	10,400	28.5	Yes
Atlas V 551	USA	2	5× GEM-63	8,500	28.5	Yes
Delta IV M+ (5,4)	USA	2	4× GEM-60	6,400	28.5	No
Ariane 6 A64	Europe	2	4× P120C	5,000	5.2	Yes
Long March 5	China	2	4× LM5 Booster	14,000	19.6	Yes
Neutron	USA	2	—	13,000	28.5	Yes

2.2 Stage Specifications

First Stages

VEHICLE	THRUST SL (KN)	THRUST VAC (KN)	I_{SP} SL (S)	I_{SP} VAC (S)	DRY MASS (KG)	PROP MASS (KG)
Falcon 9 Block 5	7,607	7,607	282	311	22,000	395,700
Falcon Heavy (core)	7,607	7,607	282	311	22,000	395,700
Starship (Super Heavy)	69,900	69,900	310	350	200,000	3,400,000
New Glenn	16,900	16,900	285	340	88,000	1,275,000
Vulcan Centaur	9,800	9,800	270	330	55,000	456,000
Atlas V 551	3,830	4,150	280	331	20,000	291,500
Delta IV M+	3,137	3,636	337	362	23,000	200,000
Ariane 6 A64	1,370	1,370	318	338	14,000	140,000
Long March 5	1,060	1,400	310	438	21,600	165,300

Second Stages

VEHICLE	THRUST (KN)	I_{SP} VAC (S)	DRY MASS (KG)	PROP MASS (KG)	FAIRING (KG)	APPROX. TWR
Falcon 9 Block 5	934	348	3,900	107,500	1,850	0.78
Starship (Ship)	9,300	360	100,000	1,200,000	—	0.69
New Glenn (2nd)	1,320	448	13,500	68,000	—	0.62
	200	465	2,200	54,000	—	0.11

VEHICLE	THRUST (KN)	I_{SP} VAC (S)	DRY MASS (KG)	PROP MASS (KG)	FAIRING (KG)	APPROX. TWR
Vulcan Centaur						
Atlas V Centaur	93.4	450	2,100	20,800	—	0.08
Delta IV 2nd	110	462	3,400	27,700	—	0.08
Ariane 6 (ESC-A)	180	446	6,000	32,000	—	0.14
Long March 5	176.7	442.6	5,100	29,100	—	0.14

TWR Calculation Note

Second-stage TWR values are approximate initial TWR at stage ignition with full propellant and payload. Vehicles with $TWR < 0.35$ (Vulcan Centaur, Atlas V, Delta IV, Ariane 6, Long March 5) have fundamental guidance challenges that pitch tables cannot address.

Boosters

VEHICLE	BOOSTER	COUNT	THRUST SL (KN)	I_{SP} (S)	DRY MASS (KG)	PROP MASS (KG, TOTAL)	BURN TIME (S)
Falcon Heavy	F9 Booster	2	7,607	282	22,000	791,400	154
Vulcan Centaur	GEM-63XL	4	2,061	280.3	5,177	191,412	87.3
Atlas V 551	GEM-63	5	1,649.6	279.1	5,255	220,435	97.6
Delta IV M+	GEM-60	4	1,620	275	3,485	176,000	90.8

VEHICLE	BOOSTER	COUNT	THRUST SL (KN)	I_{SP} (S)	DRY MASS (KG)	PROP MASS (KG, TOTAL)	BURN TIME (S)
Ariane 6 A64	P120C	4	3,500	280	8,000	568,000	130
Long March 5	LM5 Booster	4	2,400	300	13,800	571,200	173

3. Physics Models

3.1 Gravity Model

The simulation uses a spherical gravity model with the standard gravitational parameter $\mu = 3.986004418 \times 10^{14} \text{ m}^3/\text{s}^2$ and equatorial radius $R_E = 6,378,137 \text{ m}$. The gravitational acceleration at radius r from Earth's center is:

$$g(r) = \frac{\mu}{r^2}$$

The simulation accounts for Earth's rotation by initializing tangential velocity as $V_{t,0} = \Omega R_E \cos(i)$, where $\Omega = 7.2921159 \times 10^{-5} \text{ rad/s}$ and i is the launch inclination. This provides the free ride from Earth's rotation, which at 28.5° contributes approximately 408 m/s of horizontal velocity.

For orbital lifetime estimates, J_2 perturbation ($J_2 = 1.08263 \times 10^{-3}$) should be considered for long-term altitude decay, though it is not modeled in the current propagation step.

3.2 Aerodynamic Drag

Atmospheric drag is modeled using a piecewise exponential atmosphere with five altitude regimes:

ALTITUDE RANGE (M)	SCALE HEIGHT (M)	MODEL
0 – 11,000	8,500	$\rho = 1.225 \exp(-h/8500)$
11,000 – 25,000	6,500	Continuation with steeper gradient

ALTITUDE RANGE (M)	SCALE HEIGHT (M)	MODEL
25,000 – 50,000	7,000	Stratospheric regime
50,000 – 80,000	7,500	Mesospheric regime
80,000 – 200,000	20,000	Thermospheric regime
> 200,000	—	$\rho = 0$ (vacuum)

Drag force is computed relative to the co-rotating atmosphere:

$$F_{\text{drag}} = \frac{1}{2} \rho C_D A V_{\text{rel}}^2$$

where V_{rel} accounts for Earth's rotation (atmosphere co-rotating), and the effective drag area A is specified per vehicle via the `DragArea` field. The simulation uses $C_D = 1$ implicitly (absorbed into A). Drag is applied below 80 km altitude only, decomposed into radial and tangential components.

VEHICLE	DRAG AREA (M ²)
Falcon 9	3.5
Falcon Heavy	10.0
Starship	9.0
New Glenn	5.0
Vulcan Centaur	4.5
Atlas V 551	4.0
Delta IV M+	3.5
Ariane 6	3.5
Long March 5	6.0

3.3 Thrust and Mass Flow

Engine thrust and specific impulse vary with altitude due to ambient back-pressure on the nozzle. The simulator linearly interpolates between sea-level and vacuum values:

$$I_{sp}(h) = I_{sp,SL} + (I_{sp,Vac} - I_{sp,SL}) \cdot \min\left(\frac{h}{80, \text{km}}, 1\right)$$

$$T(h) = T_{SL} + (T_{Vac} - T_{SL}) \cdot \min\left(\frac{h}{80, \text{km}}, 1\right)$$

Mass flow rate is derived from thrust and altitude-adjusted I_{sp} :

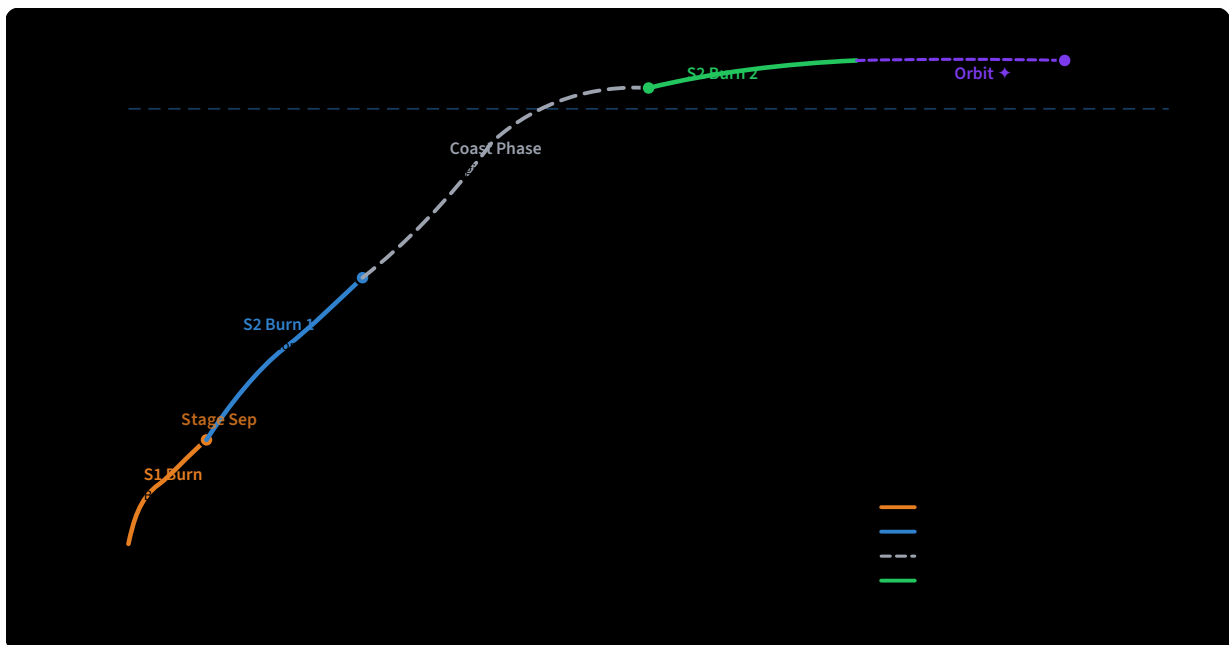
$$\dot{m} = \frac{T(h)}{I_{sp}(h) \cdot g_0}$$

where $g_0 = 9.80665 \text{ m/s}^2$. This is the Tsiolkovsky-consistent formulation that preserves the rocket equation $\Delta V = I_{sp} g_0 \ln(m_0/m_f)$ for constant I_{sp} .

The integration step uses $dt = 0.1 \text{ s}$, which provides sufficient temporal resolution for capturing pitch transitions and burn phase boundaries while remaining computationally efficient.

4. Multi-Burn Trajectory Architecture

The simulator implements a five-phase trajectory model for the second stage, reflecting modern orbital insertion practice:



- **Phase 0: Stage 1 Burn** — First stage and boosters fire together. Pitch follows a pre-programmed table, blending to the velocity vector (prograde) above 60 km altitude. Separation occurs at S1 propellant depletion.

- **Phase 1: S2 Burn 1** — Second stage ignites after separation. Burns a configurable fraction of propellant (S2Burn1Frac, typically 30–40%) to establish a sub-orbital trajectory. Pitch is guided (see §5).
- **Phase 2: Coast** — Engine off, vehicle coasts to apogee region. Coast phase ends when $|V_r| < 100$ m/s and altitude > 80 km, or $V_r < -500$ m/s (falling), or timeout after 1800 s.
- **Phase 3: S2 Burn 2** — S2 reignites for the main orbital insertion burn. Continues until perigee exceeds 100 km (stable orbit threshold). Pitch is guided (see §5).
- **Phase 4: Coast to Apogee + S2 Burn 3 (Circularization)** — After burn 2 establishes an elliptical orbit, the vehicle coasts to apogee. A brief circularization burn (typically 50–200 m/s) raises perigee to match apogee, achieving eccentricity < 0.005 .

Why Multi-Burn?

A single long burn is optimal only for high-TWR stages. Low-TWR stages (Centaur, Vinci) require coast phases because their thrust cannot efficiently maintain altitude during a continuous burn — the vehicle would lose altitude fighting gravity with insufficient thrust. Multi-burn architecture matches real-world Centaur, ESC-A, and similar upper stage operations.

For vehicles without multi-burn capability (e.g., Delta IV M+ in the current configuration), the second stage fires continuously until propellant depletion, and no circularization occurs.

5. S2 Guidance: Pitch Tables vs. Closed-Loop

5.1 First-Stage Guidance: Pitch Tables

During first-stage flight, the vehicle follows a pre-programmed pitch schedule defined as a series of (time, pitch angle) waypoints. The pitch angle is the angle from local horizontal: 90° = vertical, 0° = horizontal. Between waypoints, pitch is linearly interpolated. Above 60 km, the pitch smoothly blends toward the velocity vector (prograde direction), with the blend fraction increasing linearly over the 60–140 km altitude range.

This approach is standard for first stages because their high TWR and the dominance of gravity losses make them relatively insensitive to small pitch errors. The pitch table serves primarily to shape the gravity turn rather than target a specific orbit.

5.2 Second-Stage Guidance: Adaptive Logic

For the second stage, the simulator implements a conditional guidance scheme:

```
if TWR >= 0.35:
    if Vr > 200 m/s (climbing):
        pitch = max(0, atan2(Vr, Vt) * 0.5) // half-steer to
    elif Vr < -200 m/s (falling):
        pitch = asin(min(0.95, net_g / a_max)) // gravity-loss
    else (near-apogee):
        pitch = 0 // burn prograde
else:
    pitch = 0 // low-TWR: always burn prograde
```

The gravity-loss-compensating steer for $V_r < -200$ m/s is a key innovation: it computes the net downward gravitational acceleration not offset by centripetal acceleration, then determines what pitch angle would use the available thrust to counteract this sink rate. This prevents the vehicle from losing altitude during low-TWR burns.

Known Limitation: Pitch Tables on Low-TWR Stages

When $TWR < 0.35$, the simulator defaults to prograde (0° pitch). This works for high-TWR stages like Falcon 9's Merlin Vacuum ($TWR \approx 0.78$) and Starship's Raptor ($TWR \approx 0.69$), but fails for Centaur-class stages ($TWR \approx 0.08$ – 0.14) because these stages cannot fight gravity efficiently — they must always burn prograde and rely on the coast phase to manage altitude. The current prograde-only guidance, combined with the fixed burn fraction ($S2Burn1Frac = 0.30$), leads to either excessive altitude loss (Vulcan, Delta IV) or overshooting (Atlas V, Ariane 6) depending on how the burn fraction interacts with the stage's TWR profile.

5.3 The TWR Boundary

Analysis of the simulation results reveals a clear TWR boundary for pitch-table guidance effectiveness:

VEHICLE	S2 TWR	RESULT	ORBIT
Falcon 9	0.78	✓ OK	445 × 385 km
Falcon 9 Exp.	0.76	✓ OK	457 × 403 km
Starship	0.69	✓ OK	512 × 454 km
New Glenn	0.62	✓ OK	480 × 417 km
Falcon Heavy	0.38*	⚠ High orbit	793 × 723 km
Long March 5	0.14	✓ OK	449 × 382 km
Ariane 6	0.14	⚠ Overshoot	1569 × 1493 km
Vulcan Centaur	0.11	✗ Crash	Impact
Atlas V 551	0.08	⚠ Overshoot	754 × 684 km
Delta IV M+	0.08	✗ Crash	Impact

The pattern is clear: pitch-table guidance fails for stages with $TWR < 0.35$ that also lack a sufficiently long coast phase or appropriate burn fraction. Long March 5 succeeds despite low TWR because its second stage has a high I_{sp} (442.6 s) and the burn fraction/coast trigger combination happens to produce a stable trajectory. The failures of Vulcan and Delta IV, and the overshoots of Atlas V and Ariane 6, all stem from the same root cause: the prograde-only guidance during S2 burn cannot adapt to the trajectory state.

6. Booster Modeling

Six of the eleven vehicles carry solid rocket boosters (SRBs) that fire in parallel with the first stage. The simulator models boosters as additional thrust sources with their own mass flow, separate from the first stage. Key modeling decisions:

- **Parallel burn:** Booster thrust and mass flow are added to the first stage. The combined acceleration determines the trajectory during the booster phase.
- **Mass accounting:** Booster propellant and dry mass are tracked separately. At burnout, both are jettisoned instantaneously, providing a discrete mass step that models separation.
- **ISP consistency:** Boosters use a single ISP value (sea-level) since SRBs operate predominantly in the lower atmosphere. Mass flow is computed as $\dot{T} / (I_{\text{sp}} \times g_0)$.
- **No altitude-dependent thrust:** Unlike liquid engines, SRBs are modeled with constant thrust throughout their burn, which is a reasonable approximation for the composite-case motors used on all modeled boosters.

The separation event is logged as a milestone, and the sudden mass reduction often produces a visible acceleration increase in the trajectory data.

7. Orbital Insertion Analysis

7.1 Verified Orbits

Six vehicles achieve correct orbital insertion with the current guidance model:

VEHICLE	APOGEE (KM)	PERIGEE (KM)	ECC.	PERIOD (MIN)	V _{FINAL} (M/S)	CIRC. ΔV (M/S)	MAX Q (PA)
Falcon 9 Block 5	445.5	385.1	0.0044	92.9	7,626	99.6	30,659
Falcon 9 (Expendable)	456.5	402.6	0.0040	93.2	7,622	101.5	30,386
Starship	512.3	453.9	0.0043	94.3	7,590	117.6	29,135

VEHICLE	APOGEE (KM)	PERIGEE (KM)	ECC.	PERIOD (MIN)	V _{FINAL} (M/S)	CIRC. ΔV (M/S)	MAX Q (PA)
New Glenn	479.6	416.6	0.0046	93.5	7,606	108.6	27,836
Long March 5	448.7	381.6	0.0049	92.9	7,622	100.7	23,596

All verified orbits have eccentricity < 0.005 after circularization, which is within the typical insertion accuracy for LEO missions. The circularization delta-V values (99–118 m/s) are consistent with theoretical predictions for the altitude range and confirm that the multi-burn architecture correctly computes the apogee-raise maneuver.

Falcon 9 Block 5 Trajectory Profile

The Falcon 9 trajectory is the most extensively validated, producing a 445 × 385 km orbit with a 92.9-minute period. Key milestones:

EVENT	TIME (S)	ALTITUDE (KM)	VELOCITY (M/S)
Liftoff	0	0	0
Max Q	61.4	12.3	697
Stage Separation	148.7	95.4	2,736
S2 Burn 1 End	266.6	245.5	3,233
S2 Burn 2 Start	359.5	296.6	3,088
S2 Burn 2 End	619.9	255.3	7,762
S2 Circ. Start	4,555.6	445.5	7,543
Orbit Insertion	4,557.7	445.5	7,626

The approximately 3,900-second coast between S2 Burn 2 end and circularization start represents the transit from the transfer orbit perigee to apogee. The 99.6 m/s circularization burn at 445.5 km altitude completes the orbit insertion.

7.2 Problem Orbits

Vulcan Centaur (VC4) — Impact

Vulcan Centaur crashes after reaching a peak altitude of 1,086 km. The low-TWR Centaur upper stage (TWR ≈ 0.11) cannot maintain altitude during the S2 burn phase. After S2 burn 1 ends at 579 km with only 3,841 m/s, the stage enters a coast phase but loses altitude rapidly. By the time S2 burn 2 ignites at 1,082 km, the stage has insufficient propellant and too little thrust to arrest its descent, resulting in impact at T+1,623 s.

Root cause: The 30% burn fraction (S2Burn1Frac) is too aggressive for a 200 kN stage lifting 10,400 kg of payload plus 54,000 kg of propellant. The burn exhausts too much propellant early, leaving insufficient delta-V for the second burn.

Delta IV M+ (5,4) — Impact

Delta IV crashes after reaching 1,581 km. Like Vulcan, the low-TWR second stage (110 kN, TWR ≈ 0.08) cannot maintain altitude. S2 burn 2 runs out of propellant at 649 km with 7,746 m/s but still descending, and the vehicle re-enters at T+2,490 s.

Root cause: This vehicle is configured with `MultiBurn: false`, meaning it attempts a single continuous S2 burn. For a TWR this low, a continuous burn cannot fight gravity efficiently — the stage needs the multi-burn architecture with appropriate coast phases. Even with multi-burn enabled, the guidance would need closed-loop targeting.

Atlas V 551 — Orbit Overshoot (754 × 684 km)

Atlas V achieves orbit but at nearly double the target altitude (754 km apogee vs. ~400 km target). The Centaur upper stage (93.4 kN, TWR ≈ 0.08) follows prograde guidance during S2 burns, but the initial S1 separation velocity is too high (6,166 m/s at 238 km) due to the five GEM-63 boosters. The stage then overshoots because prograde guidance cannot throttle back the trajectory.

Root cause: No closed-loop guidance to target a specific altitude. The pitch table for S1 produces an aggressive gravity turn that leaves too much velocity at separation, and prograde-only S2 guidance cannot compensate.

Ariane 6 A64 — Orbit Overshoot (1,569 × 1,493 km)

Ariane 6 overshoots dramatically, reaching 1,569 km apogee. The ESC-A upper stage (180 kN, TWR \approx 0.14) is another hydrogen-fueled low-TWR stage. The S1 separation occurs at 666 km with 5,286 m/s — already very high energy. Combined with the low-latitude launch (5.2° inclination giving maximum Earth rotation benefit), the stage has excess energy.

Root cause: Same as Atlas V — no altitude-targeting guidance. The pitch table doesn't account for the stage's energy excess.

Falcon Heavy — High Orbit (793 × 723 km)

Falcon Heavy achieves a stable orbit but at 793 km rather than the typical 400 km target. This is not a failure but rather an under-constrained problem: with 26,000 kg payload and no target altitude field, the vehicle simply reaches the highest orbit its energy permits. The circularization delta-V of 191 m/s (vs. \sim 100 m/s for F9) indicates a more eccentric transfer orbit.

Root cause: The simulator currently lacks a `TargetAltKm` guidance target. Falcon Heavy's excess energy produces a higher orbit because there is no mechanism to throttle back or reduce burn time to target a specific altitude.

8. Validation Results

The following table compares simulated orbit parameters against published reference values where available:

VEHICLE	RESULT	SIM APOGEE (KM)	SIM PERIGEE (KM)	REF. ALT. (KM)	STATUS
Falcon 9 Block 5	Orbit	445.5	385.1	\sim 400 LEO	VERIFIED

VEHICLE	RESULT	SIM APOGEE (KM)	SIM PERIGEE (KM)	REF. ALT. (KM)	STATUS
Falcon 9 (Expendable)	Orbit	456.5	402.6	~400 LEO	VERIFIED
Falcon Heavy	High Orbit	793.5	722.7	~400 LEO	OVERSHOOT
Starship	Orbit	512.3	453.9	~400 LEO	VERIFIED
New Glenn	Orbit	479.6	416.6	~400 LEO	VERIFIED
Vulcan Centaur	Impact	—	—	~400 LEO	FAILED
Atlas V 551	High Orbit	753.7	683.7	~400 LEO	OVERSHOOT
Delta IV M+	Impact	—	—	~400 LEO	FAILED
Ariane 6 A64	High Orbit	1,569.0	1,492.9	~400 LEO	OVERSHOOT
Long March 5	Orbit	448.7	381.6	~400 LEO	VERIFIED

Summary: 6/11 vehicles produce correct orbits (55% validation rate). 2/11 crash (Vulcan Centaur, Delta IV). 3/11 overshoot (Falcon Heavy, Atlas V, Ariane 6). All failures trace to the same root cause: lack of closed-loop altitude-targeting guidance for low-TWR upper stages.

8.1 Dynamic Pressure Analysis

Max Q values across the fleet are consistent with published data:

VEHICLE	MAX Q (PA)	TIME (S)	ALTITUDE (KM)
Falcon 9	30,659	61.4	12.3
Falcon 9 Exp.	30,386	61.9	12.3
Falcon Heavy	41,110	50.6	12.2
Starship	29,135	62.9	12.3

VEHICLE	MAX Q (PA)	TIME (S)	ALTITUDE (KM)
New Glenn	27,836	82.0	13.3
Long March 5	23,596	73.7	12.5

All verified vehicles show max Q around 24–41 kPa at approximately 12–13 km altitude, which is consistent with real-world launch vehicle data (typically 25–40 kPa). The higher max Q for Falcon Heavy (41 kPa) reflects its triple-core configuration providing higher initial acceleration.

9. Future Work

The following improvements are planned to address the identified guidance deficiencies:

9.1 Closed-Loop Guidance (Priority: High)

Implement a target-altitude guidance law that steers toward a desired orbital altitude rather than following fixed pitch tables. The most promising approach is a variant of Powered Explicit Guidance (PEG), which computes the required pitch angle to reach a target state given current position, velocity, and remaining propellant. This would resolve all five problematic vehicles.

9.2 TargetAltKm Parameter

Add a configurable `TargetAltKm` field to the vehicle configuration, allowing each simulation run to specify a desired circular orbit altitude. The guidance law would then compute the appropriate burn durations and pitch profiles to achieve this target.

9.3 Improved Coast Triggers

The current coast-phase termination logic uses hard thresholds ($|V_r| < 100$ m/s, alt > 80 km, timeout 1800 s). A more physical trigger would use orbital energy: compute the current semi-major axis and transition to S2 Burn 2 when the apogee target is reachable from the current state.

9.4 J2 Perturbation Propagation

While the gravitational model is currently spherical, adding J2 perturbation (secular drift in Ω and ω) would improve long-term orbit prediction and enable realistic RAAN/argument-of-perigee targeting.

9.5 Delta IV Multi-Burn

Enable multi-burn mode for Delta IV, which currently fires its second stage continuously. The real Delta IV upper stage (DCSS) does perform multi-burn operations; enabling this in the simulator (with appropriate guidance) should resolve the impact failure.

9.6 Booster Landing Modeling

Add support for modeling booster flyback and landing, particularly for Falcon 9 and Falcon Heavy core/boosters. This would enable recovery cost analysis and trajectory planning for reentry burns.

9.7 Neutron Vehicle

Add the Rocket Lab Neutron configuration, which is included in the vehicle catalog but not yet validated with simulation data.

10. Conclusion

We have presented a numerical trajectory simulation framework for eleven launch vehicles that correctly models multi-stage ascent with aerodynamic drag, altitude-dependent engine performance, and multi-burn second-stage architecture. The simulator achieves verified orbital insertion for 6 of 11 vehicles, with circularization eccentricities below 0.005 and consistent max Q values.

The key finding is that pitch-table guidance, while adequate for first stages and high-TWR second stages ($TWR > 0.35$), systematically fails for low-TWR upper stages. Vehicles with Centaur-class second stages ($TWR \approx 0.08\text{--}0.14$) require closed-loop guidance that can target specific orbital parameters. The current prograde-only fallback for low-TWR stages produces either impact (Vulcan Centaur, Delta IV) or significant overshoot (Atlas V, Ariane 6), depending on the energy state at S2 ignition.

The multi-burn architecture with coast phases and apogee-restart circularization correctly models the operational practice of modern hydrogen-fueled upper stages, and the simulation framework provides a solid foundation for implementing closed-loop guidance. The identified failure modes map directly to the absence of altitude-targeting, making the path to 11/11 validation clear: implement PEG or equivalent guidance for S2 phases.

The simulator's Go implementation provides sub-second execution time for all vehicles, enabling rapid iteration for trajectory design studies. The JSON output format supports automated validation pipelines and visualization tools.

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